

# Terminal Sliding Mode Control – An Overview

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**ABSTRACT** Sliding mode control (SMC) has been a very popular control technology due to its simplicity and robustness against uncertainties and disturbances since its inception more than 60 years ago. Its very foundation of stability and stabilization is built on the principle of the Lyapunov theory which ascertains asymptotic stability. In the 1990s, a novel class of SMC, called the terminal sliding mode control (TSMC), was proposed which has been studied and applied extensively, giving rise to a robust control with tunable finite-time convergence delivering fast response, high precision, and strong robustness. In recent years, interest in this particular control technology has been increasing. This paper provides an overview of the state of the art of the TSMC theory and its applications, and postulates key technical issues and future challenges.

**INDEX TERMS** Finite-time convergence, robustness, sliding mode control, stability, terminal sliding mode, variable structure systems.

## I. INTRODUCTION

Sliding mode control (SMC) has been a very popular control technology for its simplicity and robustness, and studied extensively for over 60 years and received many applications [1]–[4]. Key to the SMC is the creation of the domain of attraction around a pre-defined switching manifold (or called sliding manifold) by imposing a discontinuous control. The desired control performance expectations are built into the switching manifold upon which an ideal sliding mode is established. The discontinuous control is required to alter infinitely to enforce the ideal sliding motion.

The developments of SMC theory and applications have been well documented in seminal works such as [5]–[7]. The bulk of the SMC theories developed so far are based on the foundation of the asymptotic stability-based Lyapunov theory, especially the linear switching manifolds that specify the expected control performances. The asymptotic stability is underpinned by the Lipschitz condition for the Ordinary Differential Equations (ODEs), which guarantees the required ‘smoothness’ of functions easier for analysis and synthesis. However, it is well known that the very nature of the asymptotic stability infers that, in the evolution of system dynamics, the closer to the equilibrium, the slower the state convergence. This means that the system state would never

reach the equilibrium in finite-time. While this may not be an issue in terms of practical applications, it does mean that, if a much higher steady-state precision is required, greater control force would be required which may not be feasible if control equipment/facility is restrained.

It is well known that some non-smoothness in dynamics may be used to deliver superior performance. For example, in [8] it was shown that introducing a terminal attractor (i.e. a first-order dynamics with fractional-power) in a neural learning rule results in finite-time convergence. Inspired by this, a concept of terminal sliding mode control (TSMC), was proposed in 1992 [9] for second-order systems, and extended to deal with higher-order single-input single-output (SISO) systems [10] and a class of multi-input multi-output (MIMO) systems [11], [12]. By using the finite-time convergence property [8], TSMC can ramp up the control force to speed up the convergence. In the early 2000s, a series of theoretical breakthroughs were made, such as the nonsingular TSMC [13] to address the singularity problem in TSMC systems, the fast TSMC [14] to accelerate the convergence of the basic TSMC for higher-order SISO systems, and continuous TSMC for robot control [15]. Since then, the TSMC theory and applications have finally taken off and enjoyed a significant growth of publications concerning TSMC in recent years – over 20%

annual growth over the last five years according to Google Scholar.

This paper presents an overview of the state of the art in the TSMC theory and applications, and examines the technical issues and future challenges in the context of a broader scope of technological developments such as cyber-physical systems, artificial intelligence and network systems. It is organized as follows. Section II presents the basics of SMC theory and related technical issues. Section III introduces the basic concepts of TSMC. Section IV outlines the main TSMC design methods. Section V describes the developments of TSMC. Section VI reports the extensive applications of TSMC. Section VII presents the future challenges, and Section VIII concludes the paper.

## II. BASICS OF SMC THEORY

The SMC theory was founded in the late 1950s, led by V. I. Utkin and S. V. Emelyanov [1], to deal with control systems problems involving discontinuous control [6], [16].

The SMC theory can be formulated in the following single-input affine system setting:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{b}(\mathbf{x}, t)u + \xi(\mathbf{x}, t) \quad (1)$$

where  $\mathbf{x} \in \mathbb{R}^n$  is the state,  $u \in \mathbb{R}$  is the control, and  $\mathbf{b}(\mathbf{x}, t) \neq 0$  and  $\xi(\mathbf{x}, t) \in \mathbb{R}^n$  denotes the internal uncertainties and external disturbances. If  $\xi$  is in the range of  $\mathbf{b}$ , i.e.,  $\xi \in \mathcal{R}(\mathbf{b})$ , then the well-known invariance property of SMC holds. The simplicity lies in the following discontinuous control structure by

$$u = \begin{cases} u^+(\mathbf{x}) & \text{for } s(\mathbf{x}) > 0 \\ u^-(\mathbf{x}) & \text{for } s(\mathbf{x}) < 0 \end{cases} \quad (2)$$

which is employed to enable the switching manifold  $s = 0$  to become a terminal attractor, that is maintaining  $s = 0$ . The challenge is to embed desirable characteristics and properties into  $s = 0$ , which is normally a linear hyperplane and these days increasingly nonlinear ones.

A well-known design tool is the Lyapunov theory, by which a Lyapunov function  $V = 0.5s^2$  can be used to induce  $s(\mathbf{x}) = 0$ . A common criterion is  $\dot{V} < -\rho|s|$ ,  $\rho > 0$ , which results in  $V$  to reach zero, i.e.  $s(\mathbf{x}) = 0$  in less than the time  $\rho^{-1}|s(0)|$ . When the sliding motion on  $s = 0$  occurs, that is,

$$\dot{s} = \frac{\partial s}{\partial \mathbf{x}} \dot{\mathbf{x}} = \frac{\partial s}{\partial \mathbf{x}} (\mathbf{f}(\mathbf{x}, t) + \mathbf{b}(\mathbf{x}, t)u + \xi(\mathbf{x}, t)) \quad (3)$$

where the so-called ‘‘equivalent control’’ is induced. Assuming that  $\frac{\partial s}{\partial \mathbf{x}} \mathbf{b}(\mathbf{x}, t)$  is non-singular and noting  $\xi \in \mathcal{R}(\mathbf{b})$  in (1). Hence, one can derive a virtual control signal (the equivalent control),  $u_{eq}$  by  $\dot{s} = 0$ , giving rise to in

$$u_{eq} = - \left( \frac{\partial s}{\partial \mathbf{x}} \mathbf{b}(\mathbf{x}, t) \right)^{-1} \left( \frac{\partial s}{\partial \mathbf{x}} (\mathbf{f}(\mathbf{x}, t) + \xi(\mathbf{x}, t)) \right) \quad (4)$$

In the sliding mode  $s = 0$ , the motion is governed by

$$\dot{\mathbf{x}} = \left[ \mathbf{I} - \mathbf{b}(\mathbf{x}, t) \left( \frac{\partial s}{\partial \mathbf{x}} \mathbf{b}(\mathbf{x}, t) \right)^{-1} \frac{\partial s}{\partial \mathbf{x}} \right] \mathbf{f}(\mathbf{x}, t) \quad (5)$$

which is insensitive to the matched uncertainties and disturbances  $\xi(\mathbf{x}, t)$ , dubbed as the well-known invariance property of SMC [17].

There are several typical SMC design methods, for detailed discussions on them and key challenging issues, readers are referred to [2], [6].

On-going technical issues associated with SMC include the following. While the required ideal infinite switching is beneficial, such as fast response and robustness against matched uncertainties and disturbances, it is sensitive to switching frequency if the switching frequency cannot be infinite or ideal because of hysteresis or time-delay in the controlled systems. Chattering as a direct result, if scrutinized further, may cause some irregular motions [18], though by some well-known approaches, it can be avoided [19]. For example, replacing the sign function by a boundary layer  $|s| < \epsilon$ , for a very small  $\epsilon > 0$ , within which a constant or linear control is used. Another example is to let  $\text{sgn}(s) \approx \frac{s}{|s| + \epsilon}$ , which functions a high gain control. Another new method is the high-order sliding mode (HOSM) control [20], though as shown in [7] it may result in a higher amplitude of chattering. Therefore, the chattering is still an open problem.

In SMC systems, unideal switching may induce high-frequency oscillations as well. The small time-delay in sampling (for instance, zero-order holder), and signal transmission delay in the networked control systems may also present problems.

There is another shortcoming where the sensitivity to unmodelled parts of the controlled systems due to the existence of parasitic dynamics may also induce small amplitude high-frequency oscillations. Examples include the fast actuators and sensors which are usually omitted in the control design. Last but not the least issue is the unmatched uncertainties and disturbances which destroy the invariance property of SMC. In this case, the sliding motion is dependent on the unmatched uncertainties and the disturbances.

## III. CONCEPTS OF TERMINAL SLIDING MODES

### A. BASIC TERMINAL SLIDING MODE

The concept of Terminal Sliding Mode (TSM) was rooted in a notion of terminal attractors [8] for studying the content addressable memory in neural networks. The first specific use in control design was in [9] where the following basic TSM form was used for controlling the second-order systems:

$$s = \dot{x} + \beta|x|^\lambda \text{sgn}(x) \quad (6)$$

where  $x \in \mathbb{R}$  is a variable,  $\beta > 0$ , and  $0 < \lambda < 1$ . Note that in early works on TSM, for analytical convenience,  $\lambda$  was selected as  $\lambda = q/p$  where  $p$  and  $q$  are positive odd integers. Equation (6) is more general and has been frequently used in recent times. It can be easily proved that given  $x(0) \neq 0$  and when  $s = 0$ , the dynamics (6) will reach  $x = 0$  in a finite-time specified by

$$t_s = \beta^{-1} (1 - \lambda)^{-1} |x(0)|^{1-\lambda} \quad (7)$$

The equilibrium  $x = 0$  is called a terminal attractor [8], i.e.  $x = 0$  is finite-time stable. The term “terminal” refers to the finite-time reachable equilibrium. The reaching time  $t_s$  is adjustable by using parameters  $\lambda$  and  $\beta$ .

The addition of the term  $x^\lambda$  (here we define  $x^\lambda$  as  $|x|^\lambda \text{sgn}(x)$  and both will be used interchangeably in this paper.) enhances the convergence towards the equilibrium. The closer to the equilibrium, the faster the convergence, leading to finite-time convergence. Note that while the terminal dynamics is not Lipschitz, for any  $x(0) \neq 0$ , their solution is unique in the forward time direction [21].

**B. FAST TSM AND FIXED-TIME CONVERGENCE**

It can be seen from (6) that the convergence rate of the system on the ideal sliding mode,  $s = 0$ , is inverse-proportional to the distance to the equilibrium. When  $x \gg 0$ , it does not prevail over the linear version (i.e.  $\lambda = 1$  in (6)) since the term  $x^\lambda$  in (6) would reduce the magnitude of the convergence rate at a distance of the system state to the zero. Hence, to increase the convergence rate of the basic TSM (6), a fast TSM (FTSM) was created in [14]:

$$s = \dot{x} + \alpha x + \beta |x|^\lambda \text{sgn}(x) \tag{8}$$

when  $s = 0$ , it results in  $\dot{x} = -\alpha x - \beta |x|^\lambda \text{sgn}(x)$ , which will reach  $x = 0$  in a finite-time faster than (6) for  $s = 0$  by properly choosing  $\lambda$ , i.e.

$$t_s = \alpha^{-1} (1 - \lambda)^{-1} (\ln(\alpha |x(0)|^{1-\lambda}) + \beta) - \ln \beta \tag{9}$$

Recently, extensions to (8) have been done, e.g. replacing a power-one term with a power-greater-than-one term, such that

$$s = \dot{x} + \alpha |x|^\sigma \text{sgn}(x) + \beta |x|^\lambda \text{sgn}(x) \tag{10}$$

where  $\sigma > 1$ , its reaching time can be obtained by

$$t_s = F \left( 1; \frac{\sigma - 1}{\sigma - \lambda}; \frac{2\sigma - \lambda - 1}{\sigma - \lambda}; -\beta \alpha^{-1} |x(0)|^{\lambda - \sigma} \right) \tag{11}$$

where the function  $F$  is Gaussian Hypergeometric function. Note that function  $F$  may be convergent under certain conditions, e.g. if  $\lambda \approx \sigma$ , then  $F$  is bounded above. This has been referred to as “fixed-time” convergence. Note that such control may not be practically realizable due to that the control magnitude would have to be enormous to ensure that the system state from any initial condition is brought back to the equilibrium within an initial-condition independent time-frame, though mathematically it looks elegant. A more rigorous mathematical proof of the fixed-time convergence is found in [22].

**C. NONSINGULAR TSM**

The SMC controller design requires differentiation of the sliding manifold  $s$ , which, in TSM cases, may be problematic since it may result in terms with negative powers, e.g. for (6), the time-derivative of the second term,  $\beta \lambda |x|^{\lambda-1} \text{sgn}(x) \dot{x}$ , would result in a negative-power term due to  $\lambda < 1$  causing singularity when  $x = 0$  but  $\dot{x} \neq 0$ . This leads to an infinite control, which is impossible for practical applications. To

overcome the singularity in both the basic TSM (6) and the FTSM (8), an alternative TSM, which has become very popular in recent times, is the nonsingular TSM (NTSM), defined by [13]

$$s = |\dot{x}|^\lambda \text{sgn}(\dot{x}) + \beta x \tag{12}$$

where  $1 < \lambda < 2$ , and one convenient way to express  $\lambda$  is to make it a rational number, i.e.  $\lambda = p/q$  where  $p$  and  $q$  are positive odd integers so that differentiation can be easily done. Note that for other types of  $\lambda$ , this operation can be done as well, though the expressions would become quite complex.

**D. INTEGRAL TSM**

For control systems with relative degree one, to avoid the singularity problem in the controller design of the TSMC systems, an integral TSM can be chosen as the form of [23]

$$s(t) = x(t) + \beta \int_0^t |x(\tau)|^\lambda \text{sgn}(x(\tau)) d\tau \tag{13}$$

where  $\beta$  and  $\lambda$  can be determined as the same as the method in (6). Let  $\dot{x}_1(t) = |x(t)|^\lambda \text{sgn}(x(t))$  with  $x_1(0) = -x(0)/\beta$ . In the ideal sliding motion  $s = 0$ , we have  $x(t) = -\beta x_1(t)$  or  $\dot{x}_1(t) = -\beta^\lambda |x_1(t)|^\lambda \text{sgn}(x_1(t))$ . The finite-time of  $x(t)$  from  $x(0)$  to  $x(t_f) = 0$  is the same with that of  $x_1(t)$  from  $x_1(0)$  to  $x_1(t_f) = 0$ , and can be obtained by solving the above equation:  $t_f = \beta^{-1} (1 - \lambda)^{-1} |x(0)|^{1-\lambda}$ .

**E. HIGHER-ORDER TSM**

For controlling higher-order SISO systems, the first-order TSM (6) needs to be extended to higher-order TSM as follows:

$$s = x_n + c_{n-1} \text{sgn}(x_{n-1}) |x_{n-1}|^{\alpha_{n-1}} + \dots + c_1 \text{sgn}(x_1) |x_1|^{\alpha_1} \tag{14}$$

where  $n \geq 3$ ;  $x_i = \dot{x}_{i-1}$ ,  $i = 2, \dots, n$ ,  $c_i > 0$  are selected such that the polynomial  $p^{n-1} + c_{n-1} p^{n-2} + \dots + c_2 p + c_1$ , which corresponds to (14), is Hurwitz-stable. That is, all its eigenvalues are on the left-half side of the complex plane. The parameter  $\alpha_i$  can be determined using the following conditions [24], [25]:

$$\alpha_i = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i} \quad i = 1, \dots, n - 1 \tag{15}$$

where  $\alpha_n = 1$ ,  $\alpha_{n-1} = \alpha$ ,  $\alpha \in (1 - \varepsilon, 1)$ ,  $\varepsilon \in (0, 1)$ .

**F. NESTED HIERARCHICAL TSM**

To design the higher-order TSM more conveniently, a nested hierarchical TSM was presented in [14] to achieve FTSM:

$$\begin{cases} s_1 = \dot{s}_0 + \alpha_0 s_0 + \beta_0 s_0^{\lambda_0} \\ s_2 = \dot{s}_1 + \alpha_1 s_1 + \beta_1 s_1^{\lambda_1} \\ \vdots \\ s_{n-1} = \dot{s}_{n-2} + \alpha_{n-2} s_{n-2} + \beta_{n-2} s_{n-2}^{\lambda_{n-2}} \end{cases} \tag{16}$$

where  $s_0 = x_1$ ,  $\alpha_i > 0$ ,  $\beta_i > 0$ ,  $s_i^{\lambda_i} = |s_i|^{\lambda_i} \text{sgn}(s_i)$  and  $0 < \lambda_i < 1$  ( $i = 0, \dots, n - 2$ ). The same reasoning applies so that

when  $s_{n-1} = 0$  is reached,  $s_{n-2}$  will reach zero in finite-time, so will  $s_{n-3}, \dots, s_0$ . It is easy to derive that the time taken to reach  $s_0 = 0$  is  $T = \sum_{i=1}^n t_i$ , where  $t_n$  is the time to reach the TSM  $s_{n-1}$  while

$$t_i = \alpha_{i-1}^{-1} (1 - \lambda_{i-1})^{-1} (\ln(\alpha_{i-1} |s_{i-1}(t_i)|^{(1-\lambda_{i-1})}) - \ln \beta_{i-1})$$

for  $i = n - 1, \dots, 1$ , are the time from  $s_i(t_i) \neq 0$  to  $s_i(t_i + t_{i-1}) = 0$ .

The challenge is in using it for control design where the derivative of  $s_{n-1}$  would lead to a series of terms with negative fractional powers which result in singularities. This can be dealt with in the actual control design processes to be discussed in the next section. Note that the structure (16) can be easily extended to (10) and its reaching time is similar to  $T$  aforementioned.

One should note that the nested hierarchical structure actually defines a pathway for  $x$  to reach the equilibrium. Indeed, if letting  $\lambda_i = 1$ , then one gets (by using some simple computations with the Laplace operator  $p$ )  $s_{n-1} = (p + \alpha_{n-1} + \beta_{n-1}) \dots (p + \alpha_1 + \beta_1)x$ , which is a conventional linear hyperplane.

#### IV. TSMC APPROACHES

In the following, we outline the main TSMC approaches. For illustration purposes, we use the following uncertain second-order system to inform the discussions:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \hat{f}(x) + \hat{b}(x)u + \rho(t, x, u) \end{cases} \quad (17)$$

where  $\mathbf{x} = [x_1, x_2]^T \in \mathbb{R}^2$ ,  $\hat{f}(x)$  and  $\hat{b}(x) \neq 0$  are smooth scalar functions,  $u \in \mathbb{R}$  is the control.  $\rho(t, x, u) = \Delta f(x) + d(t) + \Delta b(x)u$  represents uncertainties, including the matched uncertainty  $\Delta f(x)$ , external disturbance  $d(t)$ , and uncertainty in the control gain  $\Delta b(x)$ . It can be assumed that  $|\rho(t, x, u)| \leq F(x) + \gamma|u|$ , where  $F(x) > 0$  and  $\gamma > 0$  are known with  $|\Delta b(x)| \leq \gamma < \hat{b}(x)$ .

##### A. BASIC TSMC

For the second-order system (17), a TSM is selected in the basic form (6), i.e.  $s = x_2 + \beta x_1^\lambda$ . The control is designed by  $u = u_{eq} + u_n$ , where

$$\begin{cases} u_{eq} = -\hat{b}^{-1}(x) (\hat{f}(x) + \beta \lambda |x_1|^{\lambda-1} \text{sgn}(x_1)x_2) \\ u_n = -\hat{b}^{-1}(x) \frac{F(x) + \gamma |u_{eq}| + \eta}{1 - \gamma |\hat{b}^{-1}(x)|} \text{sgn}(s) \end{cases} \quad (18)$$

where  $\eta > 0$  is a constant. It can guarantee that the system satisfies the existence condition of the sliding mode, i.e.,  $0.5ds^2/dt < -\eta|s|$ , in the vicinity of  $s = 0$ . This means that if  $s(0) \neq 0$ , the system will reach  $s = 0$  in time  $t_r < |s(0)|/\eta$ . Once  $s = 0$ , the system (17) will behave in an identical fashion, namely  $\dot{x}_1 + \beta x_1^\lambda = 0$ , where  $x_1 = 0$  is the terminal attractor of the system (1). Assume  $t_s$  is the time that is taken from  $x_1(t_r) \neq 0$  to  $x_1(t_s + t_r) = 0$ . The system in the ideal sliding mode can be described by [27]

$$x_1(t) = \begin{cases} (|x_1(t_r)|^{(1-\lambda)} - \beta(1-\lambda)t)^{\frac{1}{1-\lambda}} \text{sgn}(x_1(t_r)) & t < t_s \\ 0 & t \geq t_s \end{cases}$$

$$x_2(t) = -\beta x_1^\lambda(t) = -\beta |x_1(t)|^\lambda \text{sgn}(x_1(t)) \quad (19)$$

where the finite-time  $t_s$  can be calculated using (7), which means that, in the ideal sliding motion, both  $x_1$  and  $x_2$  converge to zero in finite-time.

The term containing  $|x_1|^{\lambda-1} \text{sgn}(x_1)x_2$  in (18) may cause a singularity when  $x_1 = 0$  while  $x_2 \neq 0$ . When  $s = 0$ , we have  $x_2 = -\beta x_1^\lambda$ . As long as  $1/2 < \lambda < 1$ ,  $|x_1|^{\lambda-1} \text{sgn}(x_1)x_2$  is equivalent to  $|x_1|^{2\lambda-1}$  which is nonsingular. Therefore, the singularity may only occur in the reaching phase when there is insufficient control to guarantee  $x_2 \neq 0$  for  $x_1 = 0$ .

##### B. NONSINGULAR TSMC (NTSMC)

Different from the original TSM (6), an NTSM is selected in the form of (12),  $s = |x_2|^\lambda \text{sgn}(x_2) + \beta x_1$ . If the sufficient condition for the existence of the sliding-mode is satisfied, then the system will reach  $s = 0$  from  $s(0) \neq 0$  within time  $t_r \leq |s(0)|/\eta$ . On  $s = 0$ , the system will behave in an ideal sliding mode, i.e.,  $|x_2|^\lambda \text{sgn}(x_2) + \beta x_1 = 0$ , where  $x_1 = 0$  is the terminal attractor of the system (7). For the NTSM chosen as (12), the control can be designed as  $u = u_{eq} + u_n$ , where

$$\begin{cases} u_{eq} = -\hat{b}^{-1}(x) (\hat{f}(x) + \beta \lambda^{-1} x_2^{2-\lambda}) \\ u_n = -\hat{b}^{-1}(x) \frac{F(x) + \gamma |u_{eq}| + \eta}{1 - \gamma |\hat{b}^{-1}(x)|} \text{sgn}(s) \end{cases} \quad (20)$$

where  $1 < \lambda < 2$ ,  $\eta > 0$ , then the TSM (12) will be reached in finite-time. Moreover, the states  $x_1$  and  $x_2$  will converge to the equilibrium in finite-time. Different from control (18), control (20) has no singularity problem.

Another NTSMC is the saturation function-based method [27]. The TSM is chosen as the basic TSM (6), however, the control is designed as:

$$u = \hat{b}^{-1}(x) (-\hat{f}(x) + \text{sat}(u_f, u_s) - k \text{sgn}(s)) \quad (21)$$

where  $\text{sat}(u_f, u_s)$  is the saturation function,  $u_s > 0$  is a threshold,  $\eta > 0$  is a design constant, and  $u_f = -\beta \lambda x_1^{\lambda-1} x_2$ ,  $k = \frac{F(x) + \gamma |u_{eq}| + \eta}{1 - \gamma |\hat{b}^{-1}(x)|}$ .

In [27], it was proved that the TSMC strategy (21) can avoid the singularity as well as guarantee the finite-time reachability to the TSM and the finite-time convergence towards the equilibrium along the TSM  $s = 0$ .

##### C. FULL-ORDER TSMC

If the system (17) has no uncertainty in the control gain, i.e.  $\Delta b(x) = 0$ , the full-order TSMC can be utilized to attenuate the chattering. In this case, the uncertainty  $\rho(t, x, u)$  in (17) becomes  $\rho(t, x)$  due to  $\Delta b(x) = 0$  and can be assumed to satisfy

$$|\rho(t, x)| \leq F(x), \quad |\dot{\rho}(t, x)| \leq D(x) \quad (22)$$

where  $F(x)$  and  $D(x)$  are two known positive functions.

A full-order TSMC was proposed in [28] to resolve two problems, the singularity in TSMC and the chattering in both the conventional SMC and the TSMC. The full-order TSM can be expressed as

$$g = \dot{x}_2 + c_2 |x_2|^{\alpha_2} \text{sgn}(x_2) + c_1 |x_1|^{\alpha_1} \text{sgn}(x_1) \quad (23)$$



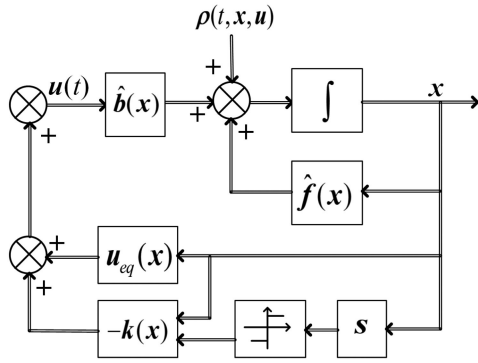


FIGURE 1. The block diagram of a system with the basic TSMC and the NTSMC.

where  $c_i$  and  $\alpha_i$  ( $i = 1, 2$ ) are constants.  $c_i$  can be selected such that  $p^2 + c_2p + c_1$  is Hurwitz.  $\alpha_i$  can be determined as  $\alpha_1 = \alpha/(2 - \alpha)$ ,  $\alpha_2 = \alpha$ , where  $\alpha \in (1 - \varepsilon, 1)$ ,  $\varepsilon \in (0, 1)$ . Once the sliding-mode  $g = 0$  is reached, the system (17) will behave in an identical fashion, that is

$$\ddot{x}_1 + c_2|\dot{x}_1|^{\alpha_2}\text{sgn}(\dot{x}_1) + c_1|x_1|^{\alpha_1}\text{sgn}(x_1) = 0 \quad (24)$$

which is finite-time convergent. The control  $u = u_{eq} + u_n$  can be set up as:

$$\begin{cases} u_{eq} = -\hat{b}^{-1}(\hat{f}(x) + c_2|x_2|^{\alpha_2}\text{sgn}(x_2) + c_1|x_1|^{\alpha_1}\text{sgn}(x_1)) \\ u_n = -\hat{b}^{-1} \int_0^t (D(x) + \eta) \text{sgn}(g) dt \end{cases} \quad (25)$$

where  $\eta$  is a positive constant.

Since no terms in control (25) have negative powers, the singularities do not occur. On the other hand, the chattering can be attenuated because of the smooth control in (25). During the ideal sliding motion, the system would behave in a full-order dynamics instead of a reduced-order dynamics [29].

#### D. CONTINUOUS TSMC

For the system (17) with  $\Delta b(x) = 0$ , the continuous SMC technique can be applied [15], [30] to attenuate the chattering. If the NTSM is chosen as in (12), the continuous TSMC strategy can be designed as  $u = u_{eq} + u_n$ , where

$$\begin{cases} u_{eq} = -\hat{b}^{-1}(x) (\hat{f}(x) + \beta\lambda^{-1}x_2^{2-\lambda}) \\ u_n = -\hat{b}^{-1}(x) (k_1s + k_2|s|^\rho \text{sgn}(s)) \end{cases} \quad (26)$$

where  $k_1, k_2 > 0$ ,  $0 < \rho < 1$ .

In the control (26), different with  $\text{sgn}(s)$ ,  $|s|^\rho \text{sgn}(s)$  is continuous. Therefore, the control (26) is continuous, and the chattering can be attenuated. The continuous TSMC approach can guarantee the finite-time convergence of the systems with  $H_2$  norm-bounded uncertainties, i.e. in (17) with  $\Delta b(x) = 0$ ,  $|\rho(t, x)| \leq F(x)$ , and  $F(x) = 0$  if  $x = 0$ , where  $F(x)$  is a positive function.

#### E. SUMMARY OF TSMC APPROACHES

The block diagrams of systems with the basic TSMC and the NTSMC, as well as the full-order TSMC are shown in Figs. 1

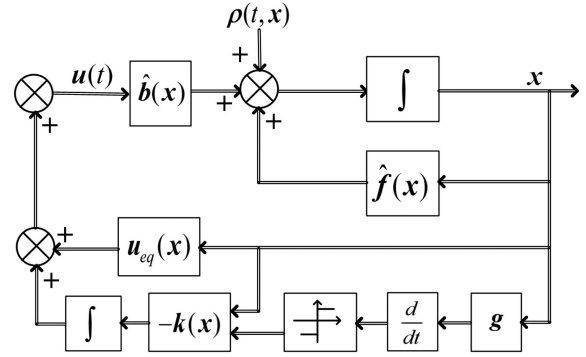


FIGURE 2. The block diagram of a system with the full-order TSMC.

	Issues	Control signal	Control gain	Sliding mode
TSMC 1994	singularity chattering uncertain gain	switching	invariant	measurable calculatable
NTSMC 2002	singularity chattering uncertain gain	switching	invariant	measurable calculatable
CTSMC 2014	singularity chattering uncertain gain	continuous	invariant	inmeasurable incalculatable
ATSMC 2019	disturbance observer, uncertain gain	continuous	adaptive	inmeasurable incalculatable

FIGURE 3. Development of the TSMC.

and 2 respectively. The main features of the TSMC can be illustrated in the following:

- 1) The sliding manifolds of the TSMC systems are non-linear, while that of the conventional SMC systems are linear.
- 2) Different from the asymptotic convergence of the conventional SMC systems, the TSMC systems are finite-time convergent.
- 3) The steady-state errors of TSMC systems are smaller than that of conventional SMC systems with similar parameter conditions. The former are  $|x_1(\infty)| < (\varphi/\beta)^{p/q}$ ,  $|x_2(\infty)| < 2\varphi$ , the latter are  $|x_1(\infty)| < \varphi/\beta$ ,  $|x_2(\infty)| < 2\varphi$ , where  $\varphi$  is the width of the boundary layer [27].
- 4) The full-order TSMC can resolve the two problems hindering the practical applications of the SMC, the singularity and the chattering simultaneously.
- 5) The continuous TSMC can attenuate the chattering and guarantee the finite-time convergence of the systems with  $H_2$  norm-bounded uncertainties.

The development of the TSMC can be shown in Fig. 3, from the initial TSMC, to nonsingular TSMC (NTSMC), continuous TSMC, and adaptive TSMC (ATSMC). The ATSMC includes the gain adaptive TSMC [29], [31], [32], adaptive sliding mode disturbance observer-based TSMC [33], adaptive TSMC [34], [35], adaptive integral TSMC [36] [37], etc.

It should be noted that the TSMC has been applied to guarantee the global fixed-time stability of systems [38].

Some SMC concepts similar to TSMC have also appeared in the control and observation, demonstrating superior performances in their particular application areas. The super-twisting SMC is an important SMC approach. It is the conservation of the features of SMC, while attenuating chattering effect in control signal. The structure of the super-twisting SMC is TSM alike and expressed by [7]

$$\begin{cases} \dot{u} = -\alpha|\sigma|^{1/2}\text{sgn}(\sigma)+v \\ \dot{v} = -M_0\text{sgn}(\sigma) \end{cases} \quad (27)$$

Levant's differentiator is a real-time differentiation algorithm to estimate derivatives of signals using sliding mode control theory. The  $k$ th-order differentiator ( $k = 1, 2, \dots$ ) utilises TSM-like output injection [20]:

$$\begin{cases} \dot{z}_0 = -\lambda_k L^{\frac{1}{k+1}} |z_0 - f(t)|^{\frac{k}{k+1}} \text{sgn}(z_0 - f(t)) + z_1 \\ \dot{z}_1 = -\lambda_{k-1} L^{\frac{1}{k}} |z_1 - v_0|^{(k-1)/k} \text{sgn}(z_1 - v_0) + z_2 \\ \vdots \\ \dot{z}_{k-1} = -\lambda_1 L^{\frac{1}{2}} |z_{k-1} - v_{k-2}|^{\frac{1}{2}} \text{sgn}(z_{k-1} - v_{k-2}) + z_k \\ \dot{z}_k = -\lambda_0 L \text{sgn}(z_k - v_{k-1}) \end{cases} \quad (28)$$

## V. TSMC DEVELOPMENTS

In the applications of the TSMC theory and techniques, there are two main obstacles, namely, the singularity and the chattering. Both of them need to be addressed appropriately in the TSMC systems. Although some recent developments are made to attack these challenges, further researches on these issues are still needed.

### A. TSMC OF SINGLE-INPUT AND MULTI-INPUT SYSTEMS

There are several ways of extending the results for single-input and multi-input systems.

For single-input systems, the challenge is dealing with that with order higher than 2. One way is to adopt the nested hierarchical structure in (16) as a sliding mode and use the equivalent-control technique for higher-order systems in controllable canonical form:  $\dot{x}_i = x_{i+1}$  ( $i = 1, \dots, n-1$ ),  $\dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u$ . As shown in [14], if the pairing conditions  $\lambda_k > \frac{n-k-1}{n-k}$  for  $k = 0, \dots, n-1$ , are met, the singularities are avoided.

For multi-input systems, the approach is rather intuitive, for example, the multi-link robotic manipulator systems can be expressed as a vector-based second-order system where the single-input techniques can be easily applied. In [11], the multi-link robotic manipulator systems are controlled by such control mechanism. Another approach is by matrix decomposition where MIMO systems are decomposed into block-form where the single-input approaches can be applied [12]. However, this does impose restrictions on system dimensions and matrices. In [12] the single-input TSMC approach was extended to those MIMO systems which can be reformulated

into a canonical form which can be easily used for MIMO TSMC design. In [31], a full-order TSMC can be also applied for MIMO systems using the virtual control technique.

### B. INTEGRAL TSMC (ITSMC)

One particular approach called here 'integral control-based design' directly uses the TSM/FTSM/NTSM as the reaching law and the sliding mode is of the same dimension of the system to be controlled. All of these methods are called the integral TSMC (ITSMC). The general purpose of ITSMC is to avoid the singularity problem. This would result in somehow a control without discontinuous though robustness features may be compromised.

In [23], an ITSMC was proposed for output tracking control of uncertain MIMO systems with relative degree one. An ITSM is designed as  $s(t) = e(t) + \alpha \int_0^t e^{q/p}(t) dt$ , where  $e(t) = \mathbf{y}(t) - \mathbf{y}_d(t)$ ,  $\mathbf{y}_d(t)$  and  $\mathbf{y}(t)$  are a desired and practical outputs respectively. The initial value of  $s(t)$  can be set to be 0 by setting the initial value of the integral in  $s(t)$  to be  $-e(0)/\alpha$ . The system is forced to start on the TSM, and the reaching phase is eliminated. Therefore, the singular problem is avoided due to the utilization of the ITSM.

In [39], the derivative-integral TSMC technique was used to suppress the chaotic behaviors in complex and uncertain biological systems. Some simulation results illustrated that the settling time using the derivative-integral TSMC technique is around 26% of that using the high-order SMC technique, and the state errors using the derivative-integral TSMC technique is below 7.1% of that using the high-order SMC technique.

### C. DIGITISATION AND DISCRETE-TIME TSMC

While TSMC demonstrates its superior finite-time convergence performance due to the addition of fractional powers in control, when digitized, it does cause some practical issues that need to be studied. In [40], the Euler discretization of TSMC was discussed and conditions obtained to ensure the existence of discrete-time FTSM. In [41], further digitisation behaviors, such as multiple convergent fixed points were observed.

SMC in the discrete-time domain is important because that many plants are discrete or digitized. On the other hand, although most plants are continuous, almost all controllers are implemented using digital computers or processors [42]–[47]. The theories of discrete-time SMC cannot be obtained directly from their continuous-time counterparts. In [48], the applicability of TSMC in the discrete-time framework is analyzed. a nilpotent function-based discrete-time TSMC (DTSMC) approach is proposed for  $n$ th-order single-input discrete-time time-invariant system expressed by  $\mathbf{x}(k+1) = \mathbf{F}(\mathbf{x}(k), u(k))$ . The discrete-time TSM is defined by  $S_d = \{\mathbf{x}(k) \mid s_d(\mathbf{x}(k)) = 0\}$ , where  $s_d(\mathbf{x}(k))$  is a real-valued scalar function. If the system dynamics are confined to the discrete-time TSM,  $\mathbf{x}(k+1) = \mathbf{F}_c(\mathbf{x}(k))$ , then the finite-time convergence characteristics can be represented in discrete-time as  $0 = \mathbf{x}(k+k_d) = \mathbf{F}_c(\mathbf{x}(k+k_d-1))$ ,  $k_d \in \mathbb{N}$ , for all  $\mathbf{x}(k) \in S_d$ . This means that

$F_c(\cdot)$  is a nilpotent function with index  $k_d$ . Hence, the discrete-time TSM may be achieved if the sliding manifold is specified so that the system confined to the manifold is nilpotent.

The design of the DTSMC can be summarized as the following three steps: 1) the discrete-time system is transformed into a Brunowsky-like canonical form using an appropriate diffeomorphism; 2) select a sliding manifold with the desired characteristics; 3) design a SMC to force the sliding manifold at the next step to zero, i.e.  $s_d(k + 1) = 0$ .

In additions, many other papers studied the DTSMC methods and their applications in many different areas [40], [49], [47].

**D. EVENT-TRIGGERED TSMC**

To save the resources for computation and/or communication, enhance the robustness and increase the response speed of systems, event-triggered techniques can be used in combination with TSMC theory. The former can be used to update the control signals of the latter, only when it is required [50]–[53].

In [54], to address the distributed consensus problem of multiagent systems with fast finite-time convergence and with bounded disturbances in a leader-follower framework, an integral sliding mode-based event-triggered SMC strategy was developed. An integral sliding surface was designed as  $S_i(t) = x_i(t) - x_i(0) - \int_0^t \chi_i^\eta(t) dt$ ,  $i = 1, \dots, n$ , where  $\chi_i$  is defined in [55],  $\eta \in (0.5, 1)$  is the ratio of positive odd numbers. On the ideal sliding motion, the system would behave in  $\dot{x}_i(t) = \chi_i^\eta(t)$ , which is finite-time convergent. The sliding mode-based consensus protocol is designed as  $u_i(t) = \chi_i^\eta(t_k^i) - K_1 \text{sgn}(S_i(t_k^i)) - K_2 S_i(t_k^i)$ , where  $t \in [t_k^i, t_{k+1}^i)$ , and  $t_k^i$  is the triggering time. The event-triggering rule is chosen by  $f_i(t) = \|e_i(t)\| - \rho_i < 0$  [54]. Besides the finite-time convergence and the robustness, the event-triggered TSMC can considerably reduce the control requirements in comparison with their time-triggered approaches, which is valuable for practical applications.

**E. INTELLIGENT TSMC**

To decrease the requirements of the system dynamics, intelligent TSMC methods were studied to accurately approximate unknown system dynamics using fuzzy wavelet network [56], or adaptive learning algorithm and fuzzy logic [57].

For a  $n$ -th order nonlinear system,  $x^{(n)} = f(x) + g(x)u$ , where  $x = [x, \dot{x}, \dots, x^{(n-1)}]^T$ ,  $g(x) > 0$ ,  $u$  is the control. Define the desired state as  $x_c$  and tracking error as  $e = x - x_c$ . The error system is  $e^{(n)} = u + z_n(x) + \Delta z(x)$ , where  $z(x) = z_n(x) + \Delta z(x) = -x_c^{(n)} + (1 - 1/g(x))x^{(n)} + f(x)/g(x)$ ,  $z_n(x)$  is the nominal value of  $z(x)$ ,  $\Delta z(x)$  denotes a bounded uncertainty,  $|\Delta z(x)| \leq Z$ , where  $Z > 0$  is a constant. In [56], a TSM is designed as  $s_i = s_{i-1} + \hat{s}_{i-1}^{p_i/q_i}$ ,  $i = 1, \dots, n - 1$ . The intelligent TSMC is designed by  $u_{ic} = u_{lc} + u_{rc} = -\hat{z}_0 - \sum_{i=1}^{n-1} \lambda_i \frac{q_i d^{n-i} s_{i-2}^{2-p_i/q_i}}{p_i d t^{n-i}}$ , where  $u_{rc}$  serves as the main control, the output of the perturbed fuzzy neural network,  $\hat{z}_0$  is utilized to online approximate  $z(x)$ , and

the compensator  $u_{rc}$  is used to overcome the effect of the approximation error on system stability.

**F. DEALING WITH UNMATCHED UNCERTAINTIES**

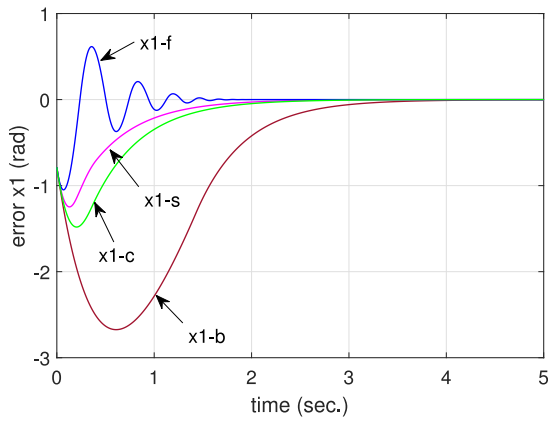
Unmatched uncertainties or disturbances have been a key problem in SMC. One way to overcome this problem is to introduce an estimator or disturbance modeler to compensate it [58]. The virtual control is another method. In [31], the TSMC can be combined with the virtual control to deal with the unmatched uncertainties or disturbances. It should be noted that no control approach can attenuate completely the unmatched uncertainties in the systems. The matched uncertainties can be attenuated completely under the condition that all system states can be controlled to zeros. While the unmatched uncertainties cannot be attenuated completely, i.e., only part of the states can be controlled to reach the equilibrium and the remaining states to converge to an area containing zero.

**G. COMPARISON OF DIFFERENT SMC METHODS**

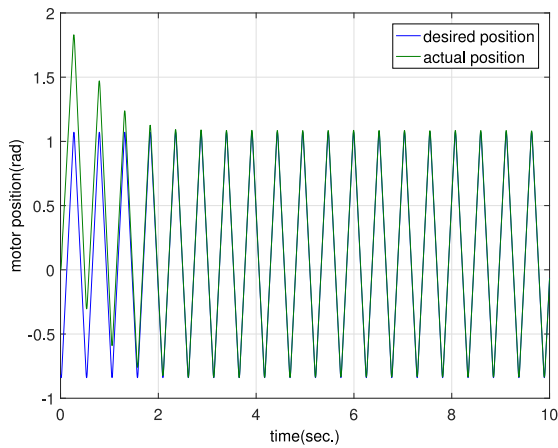
The TSMC is a special type of the SMC method and has some unique steady-state and dynamic behaviors [59]. To demonstrate the features of the TSMC, it is compared to other main SMC methods, conventional SMC and high-order SMC. Conventional SMC is related to the works on  $n$ -dimensional systems with  $m$ -dimensional control, hence the dimension of the utilized sliding manifold is  $n - m$ . The high-order SMC is generally applied for  $n$ -dimensional systems with a scalar control. Different from conventional SMC, the dimension of the sliding manifold in the high-order SMC is lower than  $n - 1$  [7].

Comparison of three different SMC methods for SISO systems is listed in Table 1. The phase portraits of three different SMC methods are shown in Fig. 8 respectively. It can be seen the differences of three different SMC methods in both the reaching phase and sliding mode.

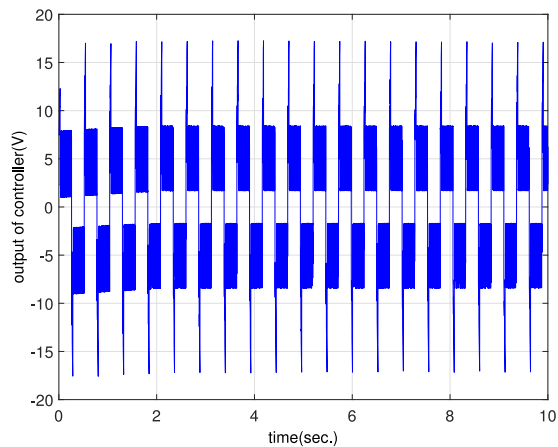
In [29], four different SMC methods were compared for the second-order DC motor system in the simulation. The position errors are shown in Fig. 4, where -b, -c, -f, and -s denote the traditional boundary layer SMC, the continuous SMC, the full-order TSMC, and the super-twisting SMC respectively. An important conclusions were drawn: the TSMC has the fastest dynamic response and smaller steady-state error. To test the effectiveness of the TSMC techniques, the real-time experiments were performed for the position control on the Quanser DC servo motor [29]. The actual position and desired position of the DC motor are shown in Fig. 5. The actual control is depicted in Fig. 6. It can be seen that the position of the DC motor can be controled to track the desired references quickly and accurately. The experimental test was carried out for the observer applying the TSMC techniques as well in [65]. The true value of SoC for a Li-ion battery, its estimation results using the proposed TSMC-based observer and the comparison with the super-twisting algorithm are depicted in Fig. 7. The former has higher accuracy compared with the latter.



**FIGURE 4.** Comparison of four continuous SMC methods on position error (-b: boundary layer; -c: continuous; -f: full-order; -s: super-twisting SMC) (Fig. 3 in [29]).



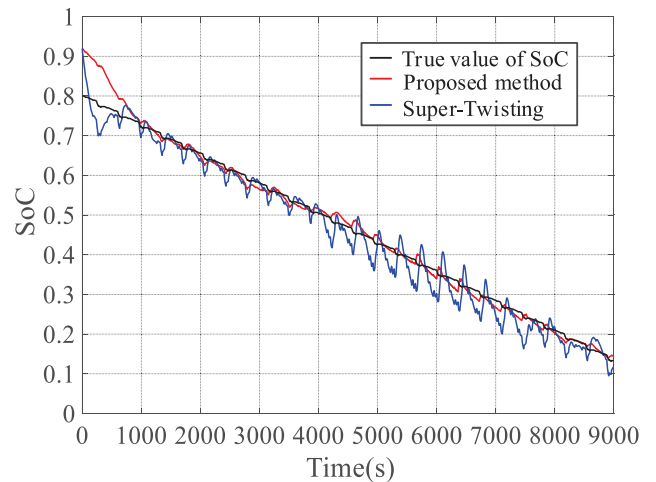
**FIGURE 5.** Practical position and desired position of the DC motor.



**FIGURE 6.** Actual control of the DC motor.

**TABLE 1.** Comparison of three different SMC methods for SISO systems

	Conventional SMC	High-order SMC	Terminal SMC
Relative degree	1	$k, k > 1$	1
Sliding manifold	linear/nonlinear	linear/nonlinear	power fractional
Dimension of sliding manifold	$n - 1$	$n - k$	$n - 1$
Convergent to sliding manifold	monotonically convergent	non-monotonically convergent	monotonically convergent
Convergence on sliding manifold	asymptotically	finite time	finite time
Chattering amplitude [7]	$1.27 \times 10^{-4}$	$4.81 \times 10^{-4}$	$1.24 \times 10^{-6}$
Controller complexity	simple	complex	complex
Robustness	strong	strong	strong
Advantages	simple, easy design and implementation, suitable for MIMO systems and observers	continuous control signal, preferable for system with output corrupted by high-level noise	finite time convergent on sliding manifold, fast dynamical response, and high tracking precision
Disadvantages	asymptotically convergence on the sliding manifold, slow convergence, large tracking error	non-monotonically convergent to the sliding manifold, large overshoot to the sliding manifold	complex calculation due to the term with fractional power, difficult for MIMO systems and observers



**FIGURE 7.** SoC estimation results using the TSMC and the super-twisting algorithm (Fig. 6 in [65]).

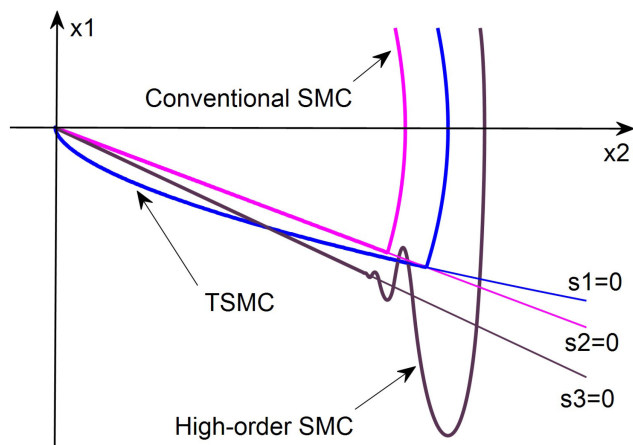
## VI. TSMC APPLICATIONS

TSMC theory and techniques have been applied extensively in recent years. In the following, we will survey its applications in different areas.

### A. STATE OBSERVATION AND PARAMETER ESTIMATION

One of the most important applications of the TSMC theories is state observation and parameter estimation. Different from the controllers, the chattering does not affect the observers. However, the output injections of the TSM observers are generally used for the parameter estimation. Therefore, a key technique is still how to soften these signals while keeping the features of SMC.





**FIGURE 8.** The phase portraits of three different SMC methods.

A TSM observer was proposed for estimating the immeasurable parameters of permanent-magnet synchronous motors (PMSMs) for mechanical systems in [60]. A second-order sliding-mode algorithm was proposed to smoothen the switching control for the observer. The experiments were carried out on a practical CNC machine tool. A similar TSM observer for a PMSM control system was developed in [26] to observe the rotor position and speed of the PMSMs. In [61], a TSM observer was applied to build a reference model on the rotor flux and then establish an online estimation method for the rotor resistance and the mutual inductance of an indirect field-oriented induction motor drive. The proposed method can be used in the case of electric vehicles or wind power generators, where the parameter deviation may degrade the drive performances.

There are some other applications on state observation and parameter estimation, such as the estimation of flux and speed of induction motors [62], the state estimation for a TCP/IP network [63], the estimation of the unmeasured states in remotely operated vehicles [64], and the estimation for state-of-charge and state-of-health of Lithium-ion batteries [65], etc. In [65], the estimation precision using the TSMC technique is 4.3%, while the estimation precision using the super-twisting algorithm is 16.13% (Fig. 6 in [65]).

## B. MOTOR AND INVERTER CONTROL

The control of motors is a challenge due to the nonlinearities and time-varying loads in the systems, especially for the high speed and high precision applications. In [66], a robust observer-controller of DC servomotors was proposed for antenna control in satellite tracking, radio telescopes, and conveyor belt systems. In [67], an integral TSMC was applied for the torque ripple reduction and overcoming of two drawbacks of SMC (the infinite-time convergence of brushless DC motors and the chattering). In [68], a fast NTSMC was developed for linear motors. Its important feature is that the control signal is inherently continuous. In [69], a TSMC was designed for the speed loop of a PMSM, which can achieve fast

convergence and better tracking precision. Further, a composite TSMC using disturbance observer was applied to reduce the chattering. In [70], a nonsingular fast TSMC strategy was proposed for a bearingless induction motor. It can not only quickly track the given speed and radial displacement simultaneously, but also enhance the operation quality. In [71], the TSMC method is utilized to regulate the speed of permanent magnet synchronous motors for achieving faster convergence, higher tracking accuracy and stronger robustness. The experimental results illustrated that compared to conventional SMC, the TSMC method has smaller speed fluctuation and the speed drop is only 58% of the conventional SMC. Furthermore, a continuous fast TSMC was developed to enhance the speed control performance of the permanent magnet synchronous motors [72].

TSMC theory and technique have also been applied in inverter control. In [73], a continuous output feedback TSMC was developed for single-phase DC-AC inverters. In [74], a recursive fast TSMC for a low-voltage microgrid system was developed and further utilized in the voltage source inverter to control the bus voltage. The particle swarm optimization can be combined with TSMC for the single-phase inverter of the electric vehicle [75] and the UPS inverters [76]. In [77], an integral compensation was added to classic TSMC to eliminate steady-state errors in the DC-AC inverter.

A DC-DC converter is a voltage converter that converts a DC voltage to another desired level of voltage. It can be divided three categories, step-down (Buck), step-up (Boost), and step-down/step-up (Buck/Boost) converters. The DC-DC converters have been widely applied in electric vehicles, battery energy storage systems, DC grid, renewable energy, DC motor drives and many other fields [78], [79], [80]. The TSMC has been used in the DC-DC converters due to its fast response and strong robustness. In [81], a fast TSMC scheme was proposed for the voltage tracking control of the DC-DC Boost converter. In [82], a finite-time disturbance observer-based TSMC approach was developed for DC-DC Buck converters. In [83], an integral-type TSMC was studied for grid-side converters used in wind energy conversion systems. The experimental results demonstrated that the TSMC approach appeared the stronger robustness, higher accuracy, and faster convergence than the conventional PI controller. In [84], a digital FTSMC approach was applied for DC-DC Buck converters and demonstrated high voltage tracking accuracy and excellent dynamic performances. In the experimental results, settling time using the FTSMC is 42.87% of that using conventional SMC, and the tracking error using the FTSMC is 15.79% of that using conventional SMC.

## C. ROBOT CONTROL AND HUMAN-ROBOT INTERACTION

The robots include rigid and flexible robots. They can be used in harmful and industrial environments. Their control is challenging due to the highly coupled nonlinear characteristics of their dynamics. In [13], a TSMC for  $n$ -link rigid robots was developed to avoid singularity and guarantee the finite-time convergence. In [85], a practical tracking control with

fractional-order TSMC using time-delay estimation (TDE) was studied for tracking control of robotic manipulators to avoid the detailed information about the robotic dynamics. In [86], [87], the TSMC technique was combined with TDE to be used for the tracking control of robotic manipulators, which can provide fast convergence, high tracking accuracy and strong robustness against parameter variations and the TDE error. In [88], a neural network-based TSMC was developed for robotic manipulators with actuator dynamics and verified using experimental results. In [89], to accurately approximate unknown dynamics, an adaptive NTSM tracking control was combined with the fuzzy wavelet network for the robot control. In [90], a TSMC combined with an unscented Kalman filter was developed, which can tolerate external disturbances.

Besides the aforementioned rigid robots, TSMC theories have been used in flexible robots. The latter is more challenging due to the nonminimum phase characteristics of flexible robots, which hinders asymptotic tracking of a desired tip trajectory with bounded control inputs. To solve this, an inverse dynamics NTSMC was proposed in [91]. The output of the flexible robot was redesigned as a function of joint angles, modes, and parameters. The system was designed based on the input-output subsystem and the zero dynamics. The former was controlled to reach its equilibrium in finite-time, and the latter was guaranteed to be asymptotically stable indirectly.

Human-robot interaction can be used for teleoperation or supervisory control, telemanipulation, robot assistants, articulated exoskeleton systems. To obtain satisfactory performances, the controller must be robust to model uncertainties and external disturbances. In [92], a modular controller using a fast TSMC was developed for articulated systems expressed by exoskeletons to perform flexion/extension movements. It can drive an upper limb exoskeleton with 3 DOF, and obtained satisfactory results. In [93], a decentralized modular control framework was proposed for robust control of walker-assisted functional electrical stimulation-activated walking. In [94], an adaptive integral TSMC was applied to an upper limb exoskeleton.

Teleoperation, also known as remote control, has many applications, such as ground-space teleoperation, telesurgery, telemanipulation, underwater robots, teleassistance robot, etc. TSMC has been used in remote control due to its superior features. In [95], to obtain a finite-time synchronization performance, a TSM-based finite-time control was developed for teleoperation system with position error constraints. In [96], a TSMC for networked bilateral teleoperation was proposed based on the TSMC technique. In [97], a fast TSM velocity observer was developed to estimate the velocity for the networked bilateral teleoperation system. In [98], a finite-time synchronization NTSMC was proposed for the master-slave control based on the adaptive fuzzy approximation. Compared with the traditional teleoperation methods, it achieves a better performance. In [64], an adaptive trajectory tracking control was developed for a remotely operated vehicle. To estimate

the unmeasured states, an adaptive TSM state observer was designed to assure the finite-time convergence of the trajectory tracking error.

#### D. MOTION AND PROCESS CONTROL

Motion control is a key technique for mechatronic systems due to the requirements on the accuracy and speed [99]. In [100], an output-based digital integral TSM predictive control scheme was developed for the precision motion control of a piezoelectric-driven positioning system. In [101], a discrete-time TSMC strategy was applied to the motion tracking control in a piezoelectric-nanopositioning system. In [102], a linear switching surface with a terminal switching surface was integrated and designed based on the precision requirement. In [103], TSMC strategies were developed to realize finite-time tracking of a DSP-based bipolar electromagnetic-levitation precise-position system. In [104], an intelligent NTSMC system was developed for the motion control of a piezo-flexural nanopositioning stage using an Elman neural network. In [105], a 3-DOF dynamic intelligent NTSMC system was proposed for the precision contours tracking of a gantry position stage. In [29], the TSMC technique was used for a class of mechatronic systems. Both the simulation and experimental results showed that the mechatronic systems had a fast response with high precision. The steady-state position error is  $9.3 \times 10^{-7}$ (rad), while that using the boundary-layer SMC is  $1.8 \times 10^{-3}$ (rad). The rising time in the former is 1.75 s, while that in the latter is 7.5 s. Besides, under the similar tracking accuracy conditions, the transient response using the TSMC is faster than that using the super-twisting algorithm.

Process control can achieve consistency, economy and safety at the production level where a fast and robust transient response is essential. In [106], a continuous NTSMC with TDE was developed for shape memory alloy actuators (SMAs). In [107], a TSMC was designed using an empirical model for the level control system to assure fast finite-time convergence and strong robustness. In [108], an output feedback TSMC framework was developed for continuous stirred tank reactor in the chemical industry. In [109], a continuous finite-time controller was developed and applied to a bioreactor process based on TSMC theory.

#### E. CONTROL OF AIRCRAFT, SPACECRAFT, AND MISSILE

Precision, response, and robustness are important to the control of aircraft, spacecraft, and missile. In [110], a TSMC was developed for a quadrotor. In [111], an adaptive compensation control strategy combining the TSMC and the input shaping method was proposed for the attitude control of a quadrotor unmanned helicopter. In [112], an adaptive NTSMC was proposed for the control of aircraft to compensate the disturbance caused by the center of gravity variation. In [113], a TSMC approach was presented for a quadrotor UAV. In [114], an integral backstepping control was combined with adaptive TSMC for the attitude control of the quadrotor. In [115], a flight TSMC was proposed for an unmanned helicopter.

The TSMC has been applied in the control of spacecraft. In [116], a TSMC was designed for spacecraft formation. In [117], a finite-time attitude tracking control scheme was proposed for spacecraft using the TSMC and Chebyshev neural network. In [118], a distributed attitude coordination TSMC was proposed for a group of spacecraft. In [119], the second-order NTSMC was combined with backstepping method for rendezvous and docking with a tumbling target spacecraft. In [120], an adaptive TSMC for the attitude control of near space hypersonic vehicles was developed. In [121], a TSMC was done for finite-time attitude synchronization of a group of spacecrafts. In [122], the TSMC technique was utilized to achieve finite-time trajectory tracking control of space manipulator. The comparison with an adaptive NN finite-time control and the PD controller demonstrated that the TSMC has the fastest convergence, highest steady-state accuracy, and least energy consumption.

The robustness and fastness abilities for missile guidance and control are essentially important. Some control methods have been developed utilizing TSMC for missile guidance and control [123]. In [124], the NTSMC method was applied to construct the composite guidance-estimation law and to guarantee the finite-time convergence of the states of the guidance dynamic system. In [125], to solve the 3D terminal guidance problem of multiple missiles cooperatively intercepting a maneuvering target, an adaptive NTSMC law was developed. In [126], an approach was proposed to integrated guidance/autopilot design for missiles on the sea using the TSMC theory in the area of the anti-vessel missile against high maneuvering targets.

#### **F. VEHICLE AND VESSEL CONTROL SYSTEMS**

The robustness is important for the control of vehicles and vessels due to external disturbances and parameter uncertainties [127]. In [128], an adaptive TSMC scheme was developed for a steer-by-wire vehicle. In [129], a hierarchical longitudinal TSMC was designed for the rear-end vehicle collision avoidance. In [130], a clutch control approach for automotive transmissions was developed using the TSMC and an observer. In [131], a counterbalancing speed control was proposed for the closed hydrostatic drive hydraulic system of the heavy vehicle under long down-slope using the TSMC to smoothly regulate the traveling speed and restrain the load disturbance. In [132], a hybrid TSMC scheme for the position-sensorless electric vehicle with a brushless DC motor was developed by combining the NTSMC with the high-order SMC.

Active automotive suspension systems can be used to improve the ride comfort and road holding. However their control is still challenging due to the highly complex-nature, serious couplings, strict constraint feasibility, substantial parameter uncertainties, and unknown external disturbance. The TSMC is a suitable and powerful tool for dealing with this control task. There are many publications on the control of active automotive suspension systems using the TSMC theory

and technique. In [133], an output feedback active suspension TSMC was developed to achieve a ride comfort while maintaining the road holding for the vehicle. In [134], a TSMC for vehicle suspension systems was proposed. In [135], an active control of a seat suspension was developed using the TSMC. In [136], a TSMC was applied for active suspension system by using a TSM manifold. The control method provided both the fast finite-time convergence and high control accuracy. The experimental results showed that the TSMC-based method could obtain the small vertical acceleration, which is 40.82% of that using active disturbance rejection controller (ADRC) and 20.52% of that using PID controller respectively.

The robustness, accuracy, and stability of measurement and control against the relative motion for a vessel on the sea is needed because it is generally waving or shifting. In [137], a disturbance estimation scheme was developed using a TSM observer to achieve formation control of multiple ocean surface vessels with high-accuracy. In [138], the NTSMC was combined with a trajectory planning to develop the surge and lateral motion controllers for the under-actuated autonomous surface vessels. In [139], a fuzzy TSMC based on multiple sliding surfaces was developed for ship course tracking steering. In [140], with the aid of the TSMC method, two asymptotically stabilizing control laws were proposed to asymptotically stabilize the underactuated surface vessel. In [141], a robust TSMC was applied to the ship control in the rudder blade deflections.

#### **G. POWER AND RENEWABLE ENERGY SYSTEMS**

To provide more robustness and faster convergence under environmental variations, TSMC theory has been applied in power systems [142]. In [143], a stabilizer based on the TSMC theory was developed for a single-machine infinite bus. In [144], a nonlinear controller was proposed for unified power flow control based on the TSMC technique for the active-reactive power and DC voltage excursions in a 3-machine 6-bus power system. In [145], a power flow controller was developed based on the TSMC technique. Compared with conventional SMC controller and a PI controller, it has the best performances. In [146], the TSMC technique was applied to enhance the power quality of wind turbines under unbalanced voltage conditions through auto-tuning the controller. The simulation results demonstrated that the magnitudes of the fluctuations in the active power (pu), DC-link voltage (V), and electromagnetic torque (pu) during the deep voltage sag conditions using the TSMC technique are 38.46%, 8.46%, and 42.86% of that respectively using conventional SMC technique.

TSMC has been applied in renewable energy systems. In [147], an integral TSMC scheme was developed for grid-connected PV arrays to maximize power extraction. In [148], an adaptive TSMC instantaneous active and reactive power control for both the rotor- and grid-side converters of the doubly fed induction generator-based wind power extraction system was developed. In [149], a TSMC method for maximum



power tracking of PV power systems was developed. In [150], a full-order TSMC for maximum power point tracking of PV cells was developed. In [151], a robust TSMC was employed to control PV system to track the maximum power point at all operating conditions. In [152], the TSMC was applied for load frequency stabilization of renewable power systems, and illustrated the superior performances against parameter uncertainties in power systems with renewable sources.

#### H. MULTIAGENT SYSTEMS

TSMC has been used in multiagent systems to solve the finite-time consensus tracking control. In [153], an NTSM protocol was developed to achieve fixed-time leader-following lag consensus of the second-order multiagent systems with input delay. In [154], the NTSMC was combined with the disturbance observer-based control to solve the finite-time leader-follower consensus problem of higher-order multi-agent systems with mismatched disturbances. In [155], a continuous nonlinear containment control protocol was constructed using an NTSMC scheme. In [156], a finite-time consensus tracking control for multi-robot systems was developed based on the TSMC technique and applied to the control of multi-robot systems with input disturbances. In [157], the nonsingular fixed-time consensus tracking was developed for second-order multiagent systems with directed topology based on the TSMC technique. In [158], the NTSMC technique was applied for a class of multi-agent tracking systems to force the tracking errors to zero in finite time.

#### I. NONHOLONOMIC SYSTEMS AND FAULT-TOLERANT CONTROL

TSMC has been applied in nonholonomic systems to achieve better performances. In [159], a recursive TSMC for tracking control of disturbed chained-form nonholonomic systems was proposed. In [160], a recursive NTSMC was applied for non-holonomic systems. In [161], an auxiliary velocity controller was integrated with an adaptive fuzzy ITSMC for a nonholonomic wheeled mobile robot. In [162], an NTSMC for wheeled mobile manipulator consisting of non-holonomic constraints was developed.

TSMC has also been applied in the fault-tolerant control. In [163], active TSMC-based fault-tolerant approaches were developed for a spacecraft. In [164], an NTSMC was designed for the rigid spacecrafts under actuator failures and saturations. In [165], a robust fault-tolerant control of spacecraft attitude stabilization in the presence of actuator failures was developed using the TSMC. In [166], a finite-time fault tolerant control was developed for uncertain robotic manipulators with actuator faults by combining the obtained fault information and the TSMC technique. In [167], an NTSMC was designed to achieve distributed cooperative attitude synchronization using consensus theory in the presence of the wheel faults. Some other methods were proposed such as adaptive fast TSMC approaches to resolve the attitude tracking control of spacecrafts subject to actuator faults and actuator saturations [168], [169]. In [170], the TSMC was combined with adaptive fuzzy PID

control to formulate an enhanced robust fault tolerant control for the purpose of enhancing the robustness and increasing the steady-state precision.

### VII. FUTURE CHALLENGES

#### A. THEORETICAL AND PRACTICAL CHALLENGES

Many TSMC theories have been proposed and applied in various applications, however, there are the following theoretical and practical challenges that need to be addressed for the development of TSMC theory and applications in the future:

- To attenuate the chattering, an important technique in conventional SMC systems is the higher-order SMC. However, when the higher-order SMC techniques are utilized to attenuate the chattering, the derivative of the control may appear infinite due to the terms with fractional powers. How to address this issue to avoid the singularities is a challenge.
- A TSM for a system is a nonlinear function of system states. Compared with conventional sliding-mode observers with linear sliding mode manifold, it is very hard to design an observer utilizing the TSMC technique due to its nonlinear features. How to design the TSM observer is another challenge.
- On the ideal sliding motion of conventional SMC systems, some states can be considered as the linear feedback of the remaining states. However, for the TSMC systems, it is nonlinear and will hinder the applications of the TSMC on the output feedback control and the MIMO systems. How to deal with this issue is a theoretical challenge as well.
- The finite-time convergence of the TSMC systems is for the continuous-time controllers. However, in practical applications, the controllers are generally discrete-time. How to keep the finite-time convergence of the TSMC systems, or to maintain this feature to the maximum is a challenge in practical aspect.
- The relationship between the parameters of the TSMC controller and the performances of the system is inherently nonlinear and quite complex. How to select the parameters of the TSMC controller using optimization algorithms is another challenge in the practical applications of the TSMC theories.
- Different from the conventional SMC, the TSM and related control strategies are nonlinear. Although the TSMC systems have good steady-state and dynamic performances, it is hard for the implementation of the TSMC controller. How to trade-off the implementation and its performances is another practical challenge as well.
- The applications of the TSMC are mainly in the controllers and observers. In most practical applications, both of them are implemented using the MCUs, DSPs or FPGAs. How to implement the TSMC controllers/observers with desirable real-time performances on such computation-resource-limited embedded devices is still a challenge.



## B. TSMC IN ARTIFICIAL INTELLIGENCE

Artificial intelligence (AI) is shaping the future technological developments. How to apply the TSMC theories in AI and how to utilize AI in TSMC design are two important issues in both the theoretical and practical aspects. The fast finite-time convergence, and high tracking precision of the TSMC can be utilized in cloud computing, big data analysis and data mining to enhance the performances, e.g. in the neural learning [171]. On the other hand, powerful AI methods and techniques can be applied to TSMC theories and applications. This is a direction which in the future will see more developments.

## C. TSMC IN NETWORK ENVIRONMENTS

Modern industrial network systems are embedded with a huge number of sensors and actuators from which vast volumes of information are available to be transmitted for coordinated control and operations via a communication network shared medium [172]. The advanced communication technologies enables cost-effective information processing and industrial control. In the same time, it encounters the problems associated with network induced delay and data packet dropout. Implementing TSMC in networked environments is a new research area and their thorough studies will be very important for future TSMC applications. Typical problems to be addressed include chattering, unmatched uncertainties and unmodelled dynamics. Furthermore, TSMC in network environments may pose new problems due to the nonsmooth nonlinearities and finite-time convergence.

## VIII. CONCLUSION

In this paper, the TSMC has been reviewed, including TSMC basics, TSMC developments, the state of the art of TSMC theory and its applications. The fundamental difference between the TSMC and conventional SMC has been investigated. The important features and advantages of TSMC have been analyzed, and the challenges in TSMC and their future trend in theory and applications have been outlined. We hope this article will help stimulate further developments and discussions in control, modelling and optimization using TSMC.

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